These expressions can be used (often with a past perfect tense) to talk about two events that happen one after another.

Note the sentence structure.

... hardly ... when / before ...
... scarcely ... when / before ...
... no sooner ... than ...

No sooner had she read the letter than she started screaming. (NOT No sooner had she read the letter when she started screaming.)

No sooner had I closed the door than someone knocked. (NOT No sooner had I closed the door before / when someone knocked.)

No sooner can come after the subject. In this case, we use the normal word order.

I had no sooner closed the door than someone knocked.

Scarcely and hardly

Scarcely had I solved one problem when / before another popped up.
Scarcely had I reached the station when the train steamed out.
Hardly had I closed my eyes when the telephone rang.

Scarcely and hardly can come after the subject. In this structure, we use the normal word order.

I had scarcely solved one problem when another popped up.
I had hardly closed my eyes when the telephone rang.

It is wrong to use than instead of when or before in this structure.

Incorrect: Hardly I had closed my eyes when the telephone rang. (Correct: Hardly had I closed...)
Incorrect: Scarcely I had stepped out when it started raining. (Correct: Scarcely had I stepped...)