Can
To express ability

Can is used to express present or general ability.

I can swim.
She can knit.
We can speak English.
Can you speak French?
She can draw beautiful pictures.

To give permission

Can is sometimes used in the sense of may to give permission. Note that many people consider this usage incorrect and prefer using may.

You can take one of those books. (More correct: You may take one of those books.)
She can do whatever she wants. (More correct: She may do whatever she wants.)
You can go now. (More correct: You may go now.)

Now-a-days can is also being increasingly used to ask permission.

While this practice is common grammarians still insist on using may for asking and giving permission.

Can I come in, Sir? (More correct: May I come in, Sir?)
Can I go to the pictures, Mom? (More correct: May I go to the pictures, Mom?)

Could

In indirect speech

Could is the past tense form of can in indirect speech.

Direct speech: She said, “I can’t climb up the hill.”
Indirect speech: She said that she couldn’t climb up the hill.
He said, “I can solve the problem.”
He said that he could solve the problem.

To talk about past ability

Could is used to talk about ability that existed in the past.

In my younger days I could run miles at a stretch.
I could swim across that river when I was young.  
She could read when she was three.  
Why couldn’t you finish the work in time?

Note that could refers to past time only when the context makes the time clear.

**To talk about possibility or uncertainty**

Could is used to talk about possibility or uncertainty. It is also common in conditional sentences.

You could win, if you worked hard. (Possibility)  
She could pass if she studied hard. (Possibility)  
I could have helped him if he had asked me.  
If I had the money I could buy a car.

**In polite requests**

Could is often used in questions expressing polite requests.

Could you wait for a minute?  
Could I lend your bicycle for a day?  
Could you, please, take me to the manager?  
Could I have a cup of coffee?